PAKISTAN SCHOOL, KINGDOM OF BAHRAIN

E – SUPPORT AND LEARNING MATERIAL

SUBJECT: MATHEMATICS **Grade: 7**

Chapter: 1 Primes, Highest Common Factor and Lowest Common Multiple

Prime Number:

A prime number is a whole number that has exactly 2 different factors, 1 and itself.

Examples:

- The factors of 3 are 3x1
- The factors of 5 are 5x1
- The factors of 29 are 29x1

Composite Number:

A composite number is a whole number that has more than 2 different factors.

Examples:

- The factors of 9 are 9x1, 3x3
- The factors of 14 are 14x1, 2x7
- The factors of 21 are 21x1, 3x7

Exercise 1 A

Q1) Determine whether each of the following is a prime or a composite number.

a). 87

Solution:

87 is an odd number, so it is not divisible by 2. Since the sum of the digits of 87 is 8 + 7 = 15 which is divisible by 3, therefore 87 is divisible by 3(divisibility test for 3). 87 is a composite number.

Homework:

Exercise 1A Q1 part **b**, **c** and **d**.

Prime Factorization:

When we show prime factorization of an integer, we list the prime factors whose product result in number

Example:

If we express 18 as a product of prime factors i.e. factors which are prime numbers

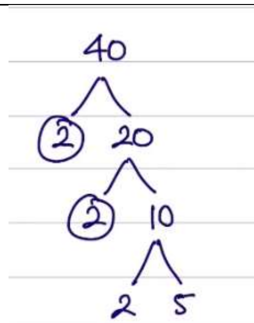
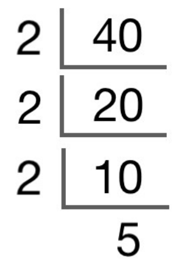
$$18 = 2 \times 3 \times 3$$
$$= 2 \times 3^2$$

We say that 2 and 3 are prime factors of 18.

The process of expressing 18 as a product of its prime factors is called the prime factorization of 18.

- Don't confuse the prime factorization of 18 with finding the factors of 18:
  18 = 1x18 = 2x9 = 3x6
  Notice that factors of 18 are 1,2,3,6,9 and 18, which are not necessarily its prime factors.

Two types of methods to solve prime factorization.

Tree Method	Up - Side Down Division
	
Start by dividing the given number by smallest prime which is 2	Now you know why it is called the upside-down division because the division symbol is literally upside down.
The factors of the numbers above are broken down into branches and indicated by the line segments.	I start the dividing the given number by the smallest prime number is 2.
We can divide 40 and its quotient by the prime number 2 three time which means prime number will have exponent of 3 in the factorization.	If that prime evenly divides a number, then I place the quotient below, continue this process as needed.
The last quotient after the repeated divisions of 2 is the prime number, which is 5.	Noted that we can perform repeated division of prime number 2, until reaching the prime number 5 as its final whole number quotient (most bottom).
Upon reaching a prime number as its last quotient in the process, this shows that we are done.	Present the final factorization as a product of exponential number having a prime number base in the exponential notation.

<u>Index Notation:</u>

Index notation is a short way of writing a number being multiplied by itself several times. The number that is being multiplied by itself is known as the "base". The number written above the base is known as the "index", "power" or "exponent". The index is the number of times that the base must be multiplied by itself.

Example:

5^3 is read as '5 to the power of 3' or simply '5 cubed'.

Exercise 1 A

Q2) Find the prime factorization of each of the following numbers, leaving your answer in index notation.

b). 187

Solution: $187 = 11 \times 17$

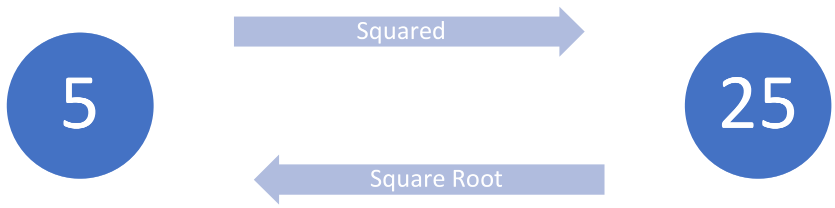
Homework:

Exercise 1A Q2 part **a**, **c** and **d**

Square Roots:

We have learnt in previous class that the area of a square with sides 5cm is 5cm x 5cm = 25cm^2. We have also learnt that if we have given a square with an area of 25cm^2, the length of its sides is $\sqrt{25}$ = 5cm, we say that the square root of 25 is 5.

$$5^2 = 5 \times 5 = 25$$



$$\sqrt{25} = \sqrt{5x5} = 5$$

Similarly,

$0 \times 0 = 0^2 = 0$ implies $\sqrt{0} = \sqrt{0x0} = 0$

$1 \times 1 = 1^2 = 1$ implies $\sqrt{1} = \sqrt{1x1} = 1$

$2 \times 2 = 2^2 = 4$ implies $\sqrt{4} = \sqrt{2x2} = 2$

$3 \times 3 = 3^2 = 9$ implies $\sqrt{9} = \sqrt{3x3} = 3$

0, 1, 4, 9 are the squares of whole numbers, they are called perfect square (square numbers).

Example: (Finding Square Root using prime factorization)

Solve $\sqrt{324}$ by prime factorization.

$324 = 2 \times 2 \times 3 \times 3 \times 3 \times 3$
$= (2 \times 3 \times 3) \times (2 \times 3 \times 3)$
$= (2 \times 3 \times 3)^2$
$\sqrt{324} = (2 \times 3 \times 3)$
$= 18$

Alternatively,

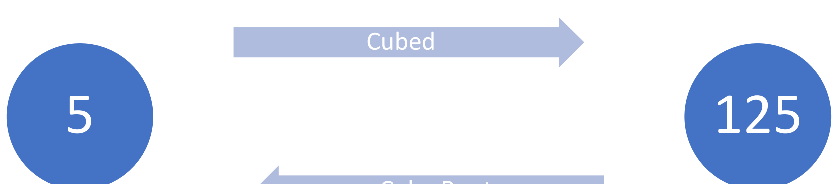
$324 = 2 \times 2 \times 3 \times 3 \times 3 \times 3$
$= 2^2 \times 3^4$
$\sqrt{324} = \sqrt{2^2} \times \sqrt{3^4}$
$= 2 \times 3^2$
$= 18$

2	324
2	162
3	81
3	27
3	9
3	3
	1

Cube Roots:

We have learnt in previous class that the volume of a cube with edges 5cm is 5cm x 5cm x 5cm = 125cm^3. We have also learnt that if we have given a cube with a volume of 125cm^2, the length of its edge is $\sqrt[3]{125}$ = 5 cm, we say that the cube root of 125 is 5.

$$5^3 = 5 \times 5 \times 5 = 125$$



$$\sqrt[3]{125} = \sqrt[3]{5 \, x \, 5 \, x \, 5} = 5$$

Similarly,

$0 \times 0 \times 0 = 0^3 = 0$ implies $\sqrt[3]{0} = \sqrt[3]{0 \times 0 \times 0} = 5$

$1 \times 1 \times 1 = 1^3 = 1$ implies $\sqrt[3]{1} = \sqrt[3]{1 \times 1 \times 1} = 1$

$2 \times 2 \times 2 = 2^3 = 8$ implies $\sqrt[3]{8} = \sqrt[3]{2 \times 2 \times 2} = 2$

$3 \times 3 \times 3 = 3^3 = 27$ implies $\sqrt[3]{27} = \sqrt[3]{3 \times 3 \times 3} = 3$

0, 1, 8, 27 are the cubes of whole numbers, they are called perfect cube (cube numbers).

Example: (Finding Cube Root using prime factorization)

Solve $\sqrt[3]{216}$ by prime factorization.

$216 = 2 \times 2 \times 2 \times 3 \times 3 \times 3$
$= (2 \times 3) \times (2 \times 3) \times (2 \times 3)$
$= (2 \times 3)^3$
$\sqrt[3]{216} = (2 \times 3)$
$= 6$

Alternatively,

$216 = 2 \times 2 \times 2 \times 3 \times 3 \times 3$
$= 2^3 \times 3^3$
$\sqrt[3]{216} = \sqrt[3]{2^3} \times \sqrt[3]{3^3}$
$= 2 \times 3$
$= 6$

2	216
2	108
2	54
3	27
3	9
3	3
	1

Exercise 1 A

Q3) Find the prime factorization of each of the following numbers, leaving your answer in index notation.

c). 3375

Solution:

$3375 = 3 \times 3 \times 3 \times 5 \times 5 \times 5$
$= (3 \times 5) \times (3 \times 5) \times (3 \times 5)$
$= (3 \times 5)^3$
$\sqrt[3]{3375} = 3 \times 5$
$= 15$

Alternatively,

$3375 = 3 \times 3 \times 3 \times 5 \times 5 \times 5$
$= 3^3 \times 5^3$
$\sqrt[3]{3375} = \sqrt[3]{3^3} \times \sqrt[3]{5^3}$
$= 3 \times 5$
$= 15$

Homework:

Exercise 1A Q3 part **a**, **b** and **d**

Q4) Given that the prime factorization of 9801 is $3^4 \times 11^2$. Find $\sqrt{9801}$ without using calculator.

Solution:

$$\sqrt{9801} = \sqrt{3^4 \times 11^2}$$
$$= 3^2 \times 11$$
$$= 99$$

Q5) Given that the prime factorization of 21952 is $2^6 \times 7^3$. Find $\sqrt[3]{21\ 952}$ without using calculator.

Solution:

$$\begin{aligned}\sqrt[3]{21\ 952} &= \sqrt[3]{2^6 \times 7^3} \\ &= 2^2 \times 7 \\ &= 28\end{aligned}$$

Q6) Estimate the value of each of following.

a) $\sqrt{66}$

Solution:

$$\begin{aligned}\sqrt{66} &\approx \sqrt{64} \\ &= 8\end{aligned}$$

b) $\sqrt{80}$

Solution:

$$\begin{aligned}\sqrt{80} &\approx \sqrt{81} \\ &= 9\end{aligned}$$

c) $\sqrt[3]{218}$

Solution:

$$\begin{aligned}\sqrt[3]{218} &\approx \sqrt[3]{216} \\ &= 6\end{aligned}$$

d) $\sqrt[3]{730}$

Solution:

$$\begin{aligned}\sqrt[3]{730} &\approx \sqrt[3]{729} \\ &= 9\end{aligned}$$

Highest Common Multiple

Definition:

The largest number that divides two or more numbers is the Highest Common Factors (H.C.F) for those numbers

Example:

Consider the numbers

30 = 2 x **3** x 5
36 = 2 x 2 x **3** x 3
42 = 2 x **3** x 7
45 = 3 x **3** x 5

3 is the largest number that divides each of these numbers, and hence, is the H.C.F for these numbers. HCF is also known as Greatest Common Factor.

Now, Teacher will discuss the two methods of finding the HCF of smaller number 18 and 30.

Method 1: Prime Factorization

Step 1: Express 18 and 30 as product of their prime factors.

Step 2: Extract the common prime factors.

Step 3: The HCF of 18 and 30 is the product of common prime factors.

Common Prime Factors

$$18 = \boxed{2} \times \boxed{3} \times 3$$
$$\underline{30 = \boxed{2} \times \boxed{3} \qquad \times 5}$$

HCF of 18 and 30 $= 2 \times 3$

$= 6$

OR

Common Prime Factors

$$18 = \boxed{2} \times \boxed{3^2}$$
$$\underline{30 = \boxed{2} \times \boxed{3} \qquad \times 5}$$

Common Factor is 3 i.e. Choose the power of 3 with the smaller index

HCF of 18 and 30 $= 2 \times 3$

$= 6$

Method 2: Division Method

We can also obtain the common prime factors by division.

2	18 , 30	←	Divide 18 and 30 by 2 to get 9 and 15
3	9 , 15	←	Divide 9 and 15 by 3 to get 3 and 5
	3 , 5	←	Stop dividing when there is no common prime Factors

HCF of 18 and 30 = 2 x 3 = 6

Example:

Find the HCF of 40, 60 and 100.

Solution:

$$40 \;= \boxed{2} \times \boxed{2} \times 2 \qquad \times \boxed{5}$$
$$60 \;= \boxed{2} \times \boxed{2} \qquad \times 3 \times \boxed{5}$$
$$\underline{100 = \boxed{2} \times \boxed{2} \qquad \times \boxed{5} \times 5}$$

HCF of 40, 60 and 100 $= 2 \times 2 \qquad \times 5$

$= 20$

Exercise 1 B

Q1) Find the Highest Common Factor of each of the following sets of numbers.

b). 84 and 156

Solution: $18 = 2^2 \times 3 \times 7$

$\underline{30 = 2^2 \times 3 \qquad \times 13}$

HCF of 84 and 156 $= 2^2 \times 3$

$= 12$

Homework:

Exercise 1B Q1 part **a**, **c** and **d**

Lowest Common Multiple:

Definition:

The lowest number which is exactly divisible by each of the given numbers is called the lowest common multiple of those numbers.

Example:

$$
\begin{aligned}
3 &= 3 \times 1 \\
31 &= 31 \times 1 \\
\underline{62} &\underline{= 2 \times 31} \\
\text{LCM} &= 2 \times 3 \times 31 \\
&= 186
\end{aligned}
$$

To find the LCM of the given numbers we express each number as a product of prime numbers that appear in prime factorization of any of the numbers, the product of these prime numbers gives us LCM. Now, Teacher will discuss the two methods of finding the LCM of any of numbers.

Method 1: Prime Factorization

Step 1: Obtain the prime factorization of each number.

Step 2: Identify the common prime factors.

Step 3: The LCM of two numbers is the product of common prime factors and all the other uncommon prime factors.

Now we can apply this method to find the LCM of 4 and 6

Common Prime Factors

$$
\begin{aligned}
4 &= \boxed{2} \times 2 \\
\underline{6} &\underline{= \boxed{2} \quad\quad \times 3} \\
\text{LCM of 4 and 6} &= 2 \times 2 \times 3 \\
&= 12
\end{aligned}
$$

Similar to finding HCF, we can obtain the common prime factors by division.

Method 2: Division Method

We can also obtain the common prime factors by division.

$$
\begin{array}{c|ccc}
\boxed{2} & 4 & , & 6 \\
\hline
 & \boxed{2 \quad , \quad 3} & &
\end{array}
$$

Divide 4 and 6 by 2 to get 2 and 3

Stop dividing when there is no common prime Factors

$$
\begin{aligned}
\text{LCM of 4 and 6} &= 2 \times 2 \times 3 \\
&= 12
\end{aligned}
$$

Example:

Lowest Common Multiple of 30 and 36

Common Prime Factors

$30 = \boxed{2} \quad \times \boxed{3} \quad \times 5$

$36 = \boxed{2} \times 2 \times \boxed{3} \times 3$

HCF of 30 and 36 $= 2 \times 2 \times 3 \times 3 \times 5$

$= 180$

OR

Common Prime Factors

$30 = \boxed{2} \times \boxed{3} \times 5$

$36 = \boxed{2^2} \times \boxed{3^2}$

HCF of 30 and 36 $= 2^2 \times 3^2 \times 5$

$= 180$

Choose the power of the Common Prime Factors 3 (i.e. 2 and 3) with the higher index and the remaining factor (i.e. 5)

Method 2: Division Method

We can also obtain the common prime factors by division.

$\boxed{2}$	30	,	36	← Divide 30 and 36 by 2 to get 15 and 18
$\boxed{3}$	15	,	18	← Divide 15 and 18 by 3 to get 5 and 6
	5	,	6	← Stop dividing when there is no common prime Factors

HCF of 30 and 36 = 2 x 3 x 5 x 6

= 180

Exercise 1 B

Q2) Find the Lowest Common Multiple of each of the following sets of numbers.

c). 12, 18 and 81

Solution:

12 = 2 x 2 x 3

18 = 2 x 3 x 3

81 = 3 x 3 x 3 x 3

LCM of 18, 30 and 40 = $\underbrace{2 \times 3}_{\text{(CPF)}} \times \underbrace{2 \times 3 \times 3 \times 3}_{\text{(RF)}}$

= 324

Homework:

Exercise 1B Q2 part **a**, **b** and **d**

Real life examples of LCM and HCF:

One of the applications of prime numbers within mathematics is to find the HCF and LCM of two or more numbers. In this Section we will solve some real-life problems involving HCF and LCM.

Example:

The lights on three light ships flash at regular intervals. The first light flashes every 18 seconds, the second light flashes every 30 seconds and the third light flashes every 40 seconds. The three lights flash together at 10 pm. At what time will they next flash together.

Solution:

$18 = \boxed{2} \times \boxed{3} \times 3$
$30 = \boxed{2} \times \boxed{3} \times \boxed{5}$
$40 = \boxed{2} \times 2 \times 2 \times \boxed{5}$

LCM of 18, 30 and 40 = 2 x 3 x 2 x 2 x 3 x 5
= 360 seconds
= 6 minutes

Therefore, the three lights will next flash together at 10:06 pm.

Exercise 1 B

Q3) Find the Largest Whole Number which is a factor of both 42 and 98.

Solution: 42 = 2 x 3 x 7
30 = 2 x 7^2

Largest whole number which is a factor of both 42 and 98 = HCF of 42 and 98
= 2 × 7
= 14

Q4) The numbers 792 and 990, written as product of their prime factors, are 792 = 2^3 x 3^2 x 11 and 990 = 2 x 3^2 x 5 x 11. Hence find the Greatest whole number that divide both 792 and 990 exactly.

Solution:

Greatest whole number that will divide both 792 and 990 exactly
= HCF of 792 and 990
= 2 x 3^2 x 11
= 198

Q5) The numbers 176 and 342, written as product of their prime factors, are 176 = 2^4 x 11 and 342 = 2 x 3^2 x 19. Hence find the Smallest whole number that is divisible by both 176 and 342.

Solution:

Smallest whole number that will divide both 176 and 342 exactly
= LCM of 176 and 342
= 2^4 x 3^2 x 11 x 19
= 30 096

Q6) Find the Smallest value of n, such that LCM of n and 15 is 45.

Solution:

15 = 3 × 5
45 = 32 × 5

Smallest value of n = 3^2
= 9

Homework:

Exercise 1B Q7 and Q8