Welcome Class 10th (arts)

Algebraic Formulas and Applications

Objectives

Students will be able to:

Simplify and rationalize the surds

4. If $x = \sqrt{5}+2$, then find the values (i) $x+\frac{1}{x}$ and (ii) $x^2+\frac{1}{x^2}$

$$x = \sqrt{5}+2$$

$$\frac{1}{x} = \frac{1}{\sqrt{5}+2} \times \frac{\sqrt{5}-2}{\sqrt{5}-2}$$

$$\frac{1}{x} = \frac{\sqrt{5}-2}{(\sqrt{5}+2)(\sqrt{5}-2)}$$

$$\frac{1}{x} = \frac{\sqrt{5}-2}{(\sqrt{5})^2-(2)^2}$$

$$\frac{1}{x} = \frac{\sqrt{5}-2}{5-4} = \sqrt{5}-2$$

$$x+\frac{1}{x} = \sqrt{5}+2+\sqrt{5}-2$$

$$x+\frac{1}{x} = 2\sqrt{5}$$

Taking square on both sides, we get

$$\left(x+\frac{1}{x}\right)^2 = \left(2\sqrt{5}\right)^2$$

$$x^2+\frac{1}{x^2}+2(x)\left(\frac{1}{x}\right) = 4(5)$$

$$x^2+\frac{1}{x^2} = 20-2$$

$$x^2+\frac{1}{x^2} = 18$$

6. If $x = \sqrt{3} - \sqrt{2}$, then find the values

(i) $x - \frac{1}{x}$ and (ii) $x^2 + \frac{1}{x^2}$

Solution:

$$x = \sqrt{3} - \sqrt{2}$$

$$\frac{1}{x} = \frac{1}{\sqrt{3} - \sqrt{2}} \times \frac{\sqrt{3} + \sqrt{2}}{\sqrt{3} + \sqrt{2}}$$

$$\frac{1}{x} = \frac{\sqrt{3} + \sqrt{2}}{(\sqrt{3} - \sqrt{2})(\sqrt{3} + \sqrt{2})}$$

$$\frac{1}{x} = \frac{\sqrt{3} + \sqrt{2}}{(\sqrt{3})^2 - (\sqrt{2})^2}$$

$$\frac{1}{x} = \frac{\sqrt{3} + \sqrt{2}}{3 - 2} = \sqrt{3} + \sqrt{2}$$

$$x - \frac{1}{x} = (\sqrt{3} - \sqrt{2}) - (\sqrt{3} + \sqrt{2})$$

$$x - \frac{1}{x} = \sqrt{3} - \sqrt{2} - \sqrt{3} - \sqrt{2}$$

$$x - \frac{1}{x} = -2\sqrt{2}$$

Taking square on both sides, we get

$$\left(x - \frac{1}{x}\right)^2 = \left(-2\sqrt{2}\right)^2$$

$$x^2 + \frac{1}{x^2} - 2(x)\left(\frac{1}{x}\right) = 4(2)$$

$$x^2 + \frac{1}{x^2} = 8 + 2$$

$$x^2 + \frac{1}{x^2} = 10$$

8. If $\frac{1}{p} = \sqrt{10} + 3$, then evaluate (i) $\left(p + \frac{1}{p}\right)^2$ (ii) $\left(p - \frac{1}{p}\right)^2$

Solution: $\frac{1}{p} = \sqrt{10} + 3$

$$p = \frac{1}{\sqrt{10}+3} \times \frac{\sqrt{10}-3}{\sqrt{10}-3}$$

$$p = \frac{\sqrt{10}-3}{\left(\sqrt{10}\right)^2 - (3)^2}$$

$$p = \frac{\sqrt{10}-3}{10-9} = \sqrt{10} - 3$$

$$p + \frac{1}{p} = \sqrt{10} - 3 + \sqrt{10} + 3$$

$$p + \frac{1}{p} = 2\sqrt{10}$$

$$\left(p + \frac{1}{p}\right)^2 = \left(2\sqrt{10}\right)^2$$

$$\left(p + \frac{1}{p}\right)^2 = 4(10) = 40$$

$$p - \frac{1}{p} = \left(\sqrt{10} - 3\right) - \left(\sqrt{10} + 3\right)$$

$$p - \frac{1}{p} = \sqrt{10} - 3 - \sqrt{10} - 3$$

$$p - \frac{1}{p} = -6$$

$$\left(p - \frac{1}{p}\right)^2 = (-6)^2$$

$$\left(p - \frac{1}{p}\right)^2 = 36$$

Q. Rationalize the denominator of the following

(i) $$\frac{b+\sqrt{b^2-a^2}}{b-\sqrt{b^2-a^2}} = \frac{b+\sqrt{b^2-a^2}}{b-\sqrt{b^2-a^2}} \times \frac{b+\sqrt{b^2-a^2}}{b+\sqrt{b^2-a^2}}$$

$$= \frac{\left[b+\sqrt{b^2-a^2}\right]^2}{\left[b-\sqrt{b^2-a^2}\right]\left[b+\sqrt{b^2-a^2}\right]}$$

$$= \frac{(b)^2+2(b)\left(\sqrt{b^2-a^2}\right)+\left(\sqrt{b^2-a^2}\right)^2}{(b)^2-\left(\sqrt{b^2-a^2}\right)}$$

$$= \frac{b^2+2b\sqrt{b^2-a^2}+b^2-a^2}{b^2-\left(b^2-a^2\right)} = \frac{2b^2-a^2+2b\sqrt{b^2-a^2}}{b^2-b^2+a^2}$$

$$= \frac{2b^2-a^2+2b\sqrt{b^2-a^2}}{a^2}$$

Activity

Q. If $x = 2 + \sqrt{3}$, then find the values

i. $x - \frac{1}{x}$

ii. $x^2 + \frac{1}{x^2}$

Solution

$$x = 2 + \sqrt{3}$$

$$\frac{1}{x} = \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}}$$

$$\frac{1}{x} = \frac{2-\sqrt{3}}{(2)^2 - \left(\sqrt{3}\right)^2}$$

$$\frac{1}{x} = \frac{2-\sqrt{3}}{4-3} = 2-\sqrt{3}$$

$$x - \frac{1}{x} = \left(2+\sqrt{3}\right) - \left(2-\sqrt{3}\right)$$

$$x - \frac{1}{x} = 2+\sqrt{3}-2+\sqrt{3}$$

$$x - \frac{1}{x} = 2\sqrt{3}$$

Taking square on both sides , we get

$$\left(x - \frac{1}{x}\right)^2 = \left(2\sqrt{3}\right)^2$$

$$x^2 + \frac{1}{x^2} - 2(x)\left(\frac{1}{x}\right) = 4(3)$$

$$x^2 + \frac{1}{x^2} = 12 + 2$$

$$x^2 + \frac{1}{x^2} = 14$$

Homework

Ex 1.3 remaining parts