Pakistan School
Kingdom of Bahrain

Welcome Class 10th (arts)

Introduction to coordinate geometry

Objectives

Students will be able to:
Use distance formula to solve triangles and collinear points

3- **Express by an equation, the fact, that the point P(x, y) is equidistant from A(2, 4) and B (6, 8).**

Solution: As the point p (x, y) is equidistant from A (2, 4) and B (6, 8) so.

$$|\overline{PA}| = |\overline{PB}|$$

$$\sqrt{(x-2)^2+(y-4)^2} = \sqrt{(x-6)^2+(y-8)^2}$$

$$\sqrt{x^2-4x+4+y^2-8y\_16} = \sqrt{x^2-12x+36+y^2-16y+364}$$

$$x^2-4x+4+y^2-8y+16 = x^2-12x+36+y^2-16y+64$$

$$-4x-8y+4+16 = -12x-16y+36+64$$

$$-4x-8y+20 = -12x+16y+100$$

$$-4x+12x-8y-16y+20-100 = 0$$

$$8x-24y-80 = 0$$

$$8(x-3y-10) = 0$$

$\Rightarrow$ $x-3y = 10$

$X-3y = 10$

Which is the required equation.

Show that the points $A(1,4)$ $B(5,6)$ and, $C(9,8)$ are collinear.

SOLUTION:

Given $A(1,4)$ $B(5,6)$ and, $C(9,8)$.

Using distance formula, we have.

$$|\overline{AB}| = \sqrt{(5-1)^2+(6-4)^2} = \sqrt{4^2+2^2} = \sqrt{20} = 2\sqrt{5}$$

$$|\overline{BC}| = \sqrt{(9-5)^2+(8-6)^2} = \sqrt{4^2+2^2} = \sqrt{20} = 2\sqrt{5}$$

$$|\overline{AC}| = \sqrt{(9-1)^2+(8-4)^2} = \sqrt{8^2+4^2} = \sqrt{80} = 4\sqrt{5}$$

Now $|\overline{AB}| + |\overline{BC}| = 2\sqrt{5} + 2\sqrt{5}$

$= 4\sqrt{5}$

$= |\overline{AC}|$

Thus, the points $A,B,$ and C are collinear.

Activity

Show that the points $A(4,3)$, $B(-2,3)$ and $B(-6,3)$ are collinear.

Solution

Given $A(4,3)$, $B(-2,3)$ and $B(-6,3)$.

Using distance formula, we have.

$$\overline{|AB|} = \sqrt{(-2-4)^2 + (3-3)^2} = \sqrt{36+0} = 6$$

$$\overline{|BC|} = \sqrt{(-6-2)^2 + (3-3)^2} = \sqrt{16+0} = 4$$

$$\overline{|AC|} = \sqrt{(-6-4)^2 + (3-3)^2} = \sqrt{100} = 10$$

$$\text{Now} \quad \overline{|AB|} + \overline{|BC|} = 6+4$$

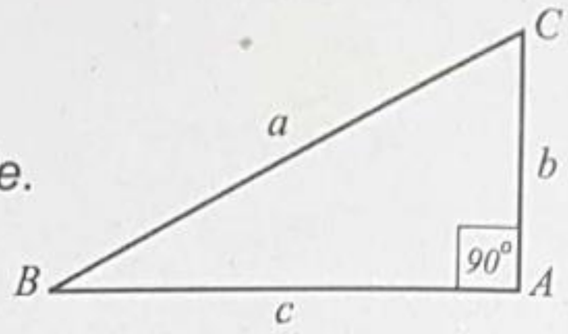
$$= 10$$

$$= \overline{|AC|}$$

Thus, the points A,B, and C are collinear.

EXAMPLE-1

Show that the points $A(-1,2)$, $B(7,5)$ and $C(2,-6)$ are vertices of a right triangle.



SOLUTION: Given $A(-1,2)$, $B(7,5)$, $C(2,-6)$.

Let a,b,c denote the lengths of the sides $\overline{BC}$, $\overline{CA}$, and $\overline{AB}$ respectively of $\Delta\, ABC$, using distance formula

$$|\overline{PQ}| = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

we have

$$a = |\overline{BC}| = \sqrt{(2-7)^2+(-6-5)^2} = \sqrt{5^2+11^2} = \sqrt{146}$$

$$b = |\overline{CA}| = \sqrt{(2-(-1))^2+(-6-2)^2} = \sqrt{3^2+8^2} = \sqrt{73}$$

$$c = |\overline{AB}| = \sqrt{(7-(-1))^2+(+5-2)^2} = \sqrt{(8)^2+(3)^2} = \sqrt{64+9} = \sqrt{73}$$

clearly $|\overline{AB}|^2 + |\overline{CA}|^2 = c^2+b^2$

$= 73 + 73 = 146 = a^2$

$= |\overline{BC}|^2$

Thus, ΔCAB, is a right triangle with right angle at A.

Activity

Show that the points A (6, 1), B(2, 7) and C(−6, −7) are vertices of a right triangle.

Solution: Here A (6, 1), B (2, 7) and C (-6, -7)

$$|\overline{AB}| = \sqrt{(2-6)^2 + (7-1)^2}$$

$$= \sqrt{(-4)^2 + (6)^2} = \sqrt{16+36}$$

$$= \sqrt{52}$$

$$|\overline{BC}| = \sqrt{(-6-2)^2 + (-7-7)^2}$$

$$= \sqrt{(-8)^2 + (-14)^2} = \sqrt{64+196}$$

$$= \sqrt{260}$$

$$|\overline{AC}| = \sqrt{(-6-6)^2 + (-7-1)^2}$$

$$= \sqrt{(-12)^2 + (-8)^2} = \sqrt{144+64}$$

$$= \sqrt{208}$$

Now $|\overline{AB}|^2 + |\overline{AC}|^2 = |\overline{BC}|^2$

$$\left(\sqrt{5^2}\right)^2 + \left(\sqrt{208}\right)^2 = \left(\sqrt{260}\right)^2$$

$$52 + 208 = 260$$

$$260 = 260$$

Homework

Ex 10.1 Q4,5,6,7