Pakistan School
Kingdom of Bahrain

WELCOME CLASS 10^{TH} (SCIENCE)

Quadratic Equations

Objectives

Students will be able to:

Solve radical equations which are convertible to quadratic equations

1.5 Radical equations

An equation involving expression under the radical sign is called a **radical equation**,

e.g., $\sqrt{x+3} = x+1$ and $\sqrt{x-1} = \sqrt{x-2}+1$

1.5 (i) Equations of the type: $\sqrt{ax+b} = cx+d$

Example 1: Solve the equation $\sqrt{3x+7} = 2x+3$.

Solution: $\sqrt{3x+7} = 2x+3$ (i)

Squaring both sides of the equation (i), we get

$$(\sqrt{3x+7})^2 = (2x+3)^2$$

or $$3x+7 = 4x^2+12x+9$$

Simplifying the above equation, we have

$$4x^2+9x+2=0$$

Applying quadratic formula,

$$x = \frac{-9 \pm \sqrt{(9)^2 - 4\times 4\times 2}}{2\times 4}$$

$$= \frac{-9 \pm \sqrt{81-32}}{8} = \frac{-9 \pm \sqrt{49}}{8} = \frac{-9 \pm 7}{8}$$

$\therefore$ $$x = \frac{-9+7}{8} = \frac{-2}{8} = \frac{-1}{4}$$

or $$x = \frac{-9-7}{8} = \frac{-16}{8} = -2$$

Note: Extraneous root is introduced either by squaring the given equation or clearing it of fractions.

Checking:

Putting $x = -\frac{1}{4}$ in the equation (i), we have

$$\sqrt{3\left(-\frac{1}{4}\right)+7} = 2\left(-\frac{1}{4}\right)+3 \Rightarrow \sqrt{\frac{-3+28}{4}} = -\frac{1}{2}+3 \Rightarrow \sqrt{\frac{25}{4}} = \frac{5}{2} \text{ which is true.}$$

Putting $x = -2$ in the equation (i), we get

$$\sqrt{3(-2)+7} = 2(-2)+3 \Rightarrow \sqrt{1} = -1 \text{ which is not true.}$$

On checking, we find that $x = -2$ does not satisfy the equation (i), so it is an extraneous root. Thus the solution set is $\left\{-\frac{1}{4}\right\}$.

Q. Solve the following equations

1. $2x + 5 = \sqrt{7x + 16}$

Squaring both sides, we get

$$(2x+5)^2 = \left(\sqrt{7x+16}\right)^2$$

$$4x^2 + 20x + 25 = 7x + 16$$

$$4x^2 + 20x + 25 - 7x - 16 = 0$$

$$4x^2 + 20x - 7x + 25 - 16 = 0$$

$$4x^2 + 13x + 9 = 0$$

$$4x^2 + 9x + 4x + 9 = 0$$

$$x(4x+9) + 1(4x+9) = 0$$

$$(x+1)(4x+9) = 0$$

$$x+1=0 \quad or \quad 4x+9=0$$

$$x=-1 \qquad 4x=-9$$

$$x = -\frac{9}{4}$$

Check :

Put $x=-1$ in eq.(i), *we get*

$$2(-1)+5 = \sqrt{7(-1)+16} \qquad \Rightarrow -2+5 = \sqrt{-7+16}$$

$$3 = \sqrt{9} \Rightarrow 3 = 3 \text{ (which is true)}$$

Put $x = -\frac{9}{4}$ *in eq*(i), *we get*

$$2\left(-\frac{9}{4}\right)+5 = \sqrt{7\left(-\frac{9}{4}\right)+16}$$

$$-\frac{9}{2}+5 = \sqrt{-\frac{63}{4}+16}$$

$$\frac{1}{2} = \sqrt{\frac{1}{4}}$$

$$\frac{1}{2} = \frac{1}{2} \text{(which is true)}$$

Thus, solution set = $\left\{-1, -\frac{9}{4}\right\}$

(5) $\sqrt{x+5}+\sqrt{x+21}=\sqrt{x+60}$

Solution:

Squaring both sides, we get

$$\left(\sqrt{x+5}+\sqrt{x+21}\right)^2=\left(\sqrt{x+60}\right)^2$$

$$(x+5)+(x+21)+2\sqrt{(x+5)\quad(x+21)}=x+60$$

$$x+5+x+21+2\sqrt{x^2+26x+105}=x+60$$

$$2x+26+2\sqrt{x^2+26x+105}=x+60$$

$$2\sqrt{x^2+26x+105}=x+60-2x-26$$

$$2\sqrt{x^2+26x+105}=-x+34$$

$$2\sqrt{x^2+26x+105}=-(x-34)$$

Squaring both sides, we get

$$\left(2\sqrt{x^2+26x+105}\right)^2=\left[-(x-34)\right]^2$$

$$4\left(x^2+26x+105\right)=x^2-68x+1156$$

$$4x^2+104x+420=x^2-68x+1156$$

$$4x^2-x^2+104x+68x+420-1156=0$$

$$3x^2+172x-736=0$$

Here a=3, b=172, c=-736

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-172\pm\sqrt{(172)^2-4(3)(-736)}}{2(3)}$$

$$x=\frac{-172\pm\sqrt{29584+8832}}{6}$$

$$x=\frac{-172\pm\sqrt{38416}}{6}$$

$$x=\frac{-172-196}{6}\quad or \quad x=\frac{-172+196}{6}$$

$$x=-\frac{368}{6}\qquad x=\frac{24}{6}$$

$$x=-\frac{184}{3}\qquad x=4$$

Check :

$$Put \qquad x = -\frac{184}{3} \text{ in eq.}(i), we\ get$$

$$\sqrt{-\frac{184}{3}+5}+\sqrt{\frac{-184}{3}+21}=\sqrt{\frac{-184}{3}+60}$$

$$\sqrt{-\frac{169}{3}}+\sqrt{-\frac{121}{3}}=\sqrt{-\frac{4}{3}} \qquad \text{(which is not true)}$$

$$Put\ x = 4\ \ in\ eq(i), we\ get$$

$$\sqrt{4+5}+\sqrt{4+21}=\sqrt{4+60}$$

$$\sqrt{9}+\sqrt{25}=\sqrt{64}$$

$$3+5=8$$

$$8=8 \qquad \text{(which is true)}$$

Thus, solution set = $\{8\}$

Activity

Q. Solve the following equation

$$4x = \sqrt{13x + 14} - 3$$

Solution

$$4x = \sqrt{13x+14} - 3 \qquad ......(i)$$

$$4x+3 = \sqrt{13x+14}$$

Squaring both sides, we get

$$(4x+3)^2 = \left(\sqrt{13x+14}\right)^2$$

$$16x^2 + 24x + 9 = 13x + 14$$

$$16x^2 + 24x - 13x + 9 - 14 = 0$$

$$16x^2 + 11x - 5 = 0$$

$$16x^2 + 16x - 5x - 5 = 0$$

$$16x(x+1) - 5(x+1) = 0$$

$$(16x-5)(x+1) = 0$$

$$\textit{Either } 16x - 5 = 0 \quad \textit{or} \quad x+1 = 0$$

$$16x = 5 \qquad x = -1$$

$$x = \frac{5}{16} \qquad x = 1$$

Solution

Check :

Put $x = \frac{5}{16}$ in eq.(i), *we get*

$$4\left(\frac{5}{16}\right) = \sqrt{13\left(\frac{5}{16}\right) + 14} - 3$$

$$\frac{5}{4} = \sqrt{\frac{65}{16} + 14} - 3$$

$$\frac{5}{4} = \sqrt{\frac{289}{16}} - 3$$

$$\frac{5}{4} = \frac{17}{4} - 3$$

$$\frac{5}{4} = \frac{5}{4} \quad \text{(which is true)}$$

Put $x = -1$ *in eq*(i), *we get*

$$4(-1) = \sqrt{13(-1) + 14} - 3$$

$$-4 = \sqrt{-13 + 14} - 3$$

$$-4 = \sqrt{1} - 3$$

$$-4 = 1 - 3$$

$$-4 \neq -2 \quad \text{(which is not true)}$$

Thus, solution set $= \left\{\frac{5}{16}\right\}$

Homework

Ex 1.4 Remaining parts