Pakistan School
Kingdom of Bahrain

WELCOME CLASS 10TH (SCIENCE)

Theory of Quadratic Equations

Objectives

Students will be able to:

Find discriminant and check the nature of roots

2.1.1 Discriminant ($b^2 - 4ac$) of the quadratic expression $ax^2 + bx + c$.

We know that two roots of the equation $ax^2 + bx + c = 0$, $a \neq 0$ (i)

are $\frac{-b + \sqrt{b^2 - 4ac}}{2a}$, and $\frac{-b - \sqrt{b^2 - 4ac}}{2a}$.

The nature of these roots depends on the value of the expression "$b^2 - 4ac$" which is called the "discriminant" of the quadratic equation (i) or the quadratic expression $ax^2 + bx + c$.

2.1.2 To find the discriminant of a given quadratic equation.

We explain the procedure to find the discriminant of a given quadratic equation through the following example:

Example 1: Find the discriminant of the following equations.

(a) $2x^2 - 7x + 1 = 0$ (b) $x^2 - 3x + 3 = 0$

Solution:

(a) $2x^2 - 7x + 1 = 0$

Here $a = 2,\ b = -7,\ c = 1$

Disc. $= b^2 - 4ac$

$= (-7)^2 - 4(2)(1)$

$= 49 - 8 = 41$

(b) $x^2 - 3x + 3 = 0$

Here $a = 1, b = -3, c = 3$

Disc. $= b^2 - 4ac$

$= (-3)^2 - 4(1)(3)$

$= 9 - 12 = -3$

2.1.3 Nature of the roots of a quadratic equation through discriminant.

The roots of the quadratic equation $ax^2 + bx + c = 0$, $(a \neq 0)$ are $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ and its discriminant is $b^2 - 4ac$.

When a, b and c are rational numbers.

(i) If $b^2 - 4ac > 0$ and is a perfect square, then the roots are rational (real) and unequal.

(ii) If $b^2 - 4ae > 0$ and is not a perfect square, then the roots are irrational (real) and unequal.

(iii) If $b^2 - 4ac = 0$, then the roots are rational (real) and equal.

(iv) If $b^2 - 4ac < 0$, then the roots are imaginary (complex conjugates).

1. Find the discriminant of the following given quadratic equation:

$(i)\ 2x^2+3x-1=0$

Solution:

$2x^2+3x-1=0$

Compare it with

$ax^2+bx+c=0$

Here $a=2, b=3, c=-1$

Disc. $=b^2-4ac$

$=(3)^2-4(2)(-1)$

$=9+8$

$=17$

$(iv)\ 4x^2-7x-2=0$

Solution :

$4x^2-7x-2=0$

Compare it with

$ax^2+bx+c=0$

Here $a=4, b=-7, c=-2$

Disc. $=b^2-4ac$

$=(-7)^2-4(4)(-2)$

$=49+32$

$=81$

2. Find the nature of the roots of the following given quadratic equations and verify the result by solving the equations:

$(i)\ x^2 - 23x + 120 = 0$

Solution :

$x^2 - 23x + 120 = 0$

Compare it with

$ax^2 + bx + c = 0$

Here $a = 1, b = -23, c = 120$

Disc. $= b^2 - 4ac$

$= (-23)^2 - 4(1)(120)$

$= 529 - 480$

$= 49$

$= (7)^2 > 0$

As the disc. is positive and is a perect square.

The roots are rational (real) and unequal

Verification

$x^2 - 23x + 120 = 0$

$x^2 - 15x - 8x + 120 = 0$

$x(x-15) - 8(x-15) = 0$

$(x-8)(x-15) = 0$

Either $x - 8 = 0$ *or* $x - 15 = 0$

$x = 8$ $\qquad$ $x = 15$

Thus, the roots are rational (real) and unequal.

$(iii)\ 16x^2-24x+9=0$

Solution :

$16x^2-24x+9=0$

Compare it with

$ax^2+bx+c=0$

Here $a=16,\ b=-24,\ c=9$

Disc. $=b^2-4ac$

$=(-24)^2-4(16)(9)$

$=576-576$

$=0$

As the disc. is zero.

Thereor the roots are real and equal.

Verification

$16x^2-24x+9=0$

Using quadratic formula

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{-(-24)\pm\sqrt{(-24)^2-4(16)(9)}}{2(16)}$$

$$=\frac{24\pm\sqrt{576-576}}{32}$$

$$=\frac{24\pm\sqrt{0}}{32}$$

$$=\frac{24}{32}=\frac{3}{4}$$

Thus, the roots are real and equal.

Activity

Q. Find the nature of the roots of the following given quadratic equations and verify the result by solving the equations:

$$3x^2 + 7x - 13 = 0$$

Solution

$3x^2+7x-13=0$

Compare it with

$ax^2+bx+c=0$

Here $a=3, b=7, c=-13$

Disc. $=b^2-4ac$

$=(7)^2-4(3)(-13)$

$=49+156$

$=205>0$

As the disc. is positive and not a perfect square.

Thereor the roots are irrational $(real)$ and equal.

Verification

$3x^2+7x-13=0$

Using quadratic formula

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{-7\pm\sqrt{(7)^2-4(3)(-13)}}{2(3)}$$

$$=\frac{-7\pm\sqrt{49+156}}{6}$$

$$=\frac{-7\pm\sqrt{205}}{6}$$

Thus, the roots are irrational (real) and unequal.

Homework

Ex 2.1 Remaining parts