Pakistan School
Kingdom of Bahrain

WELCOME CLASS 10TH (SCIENCE)

Theory of Quadratic Equations

Objectives

Students will be able to:

Solve questions related to discriminant

7. If the roots of the equation $\left(c^2-ab\right)x^2-2\left(a^2-bc\right)x+\left(b^2-ac\right)=0$ are equal, then $a=0\ or\ a^3+b^3+c^3=3abc$

Solution:

$\left(c^2-ab\right)x^2-2\left(a^2-bc\right)x+\left(b^2-ac\right)=0$

Here $a=c^2-ab,\ b=-2\left(a^2-bc\right),\ c=b^2-ac$

As the roots are equal, So

$Disc.=0$

$b^2-4ac=0$

$\left[-2\left(a^2-bc\right)\right]^2-4\left(c^2-ab\right)\left(b^2-ac\right)=0$

$4\left[\left(a^2-2a^2bc+b^2c^2\right)-\left(b^2c^2-ac^3-ab^3+a^2bc\right)\right]=0$

$a^4-2a^2bc+b^2c^2-b^2c^2+ac^3+ab^3-a^2bc=0$

$\Rightarrow a^4+ab^3+ac^3-3a^2bc=0$

$a\left(a^3+b^3+c^3-3abc\right)=0$

Either a=0 or $a^3+b^3+c^3-3abc$

Hence proved.

Activity

Q. Show that the roots of the given equation are rational.

$a(b-c)x^2 + b(c-a)x + c(a-b) = 0$

Solution

$$a(b-c)x^2+b(c-a)x+c(a-b)=0$$

Here $a=a(b-c),\ b=b(c-a),\ c=c(a-b)$

Disc.

$$=b^2-4ac$$

$$=[b(c-a)]^2-4[a(b-c)][c(a-b)]$$

$$=b^2(c-a)^2-4ac(ab-b^2-ac+bc)$$

$$=b^2c^2+a^2b^2-2ab^2c-4a^2bc+4ab^2c+4a^2c^2-4abc^2$$

$$=a^2b^2+b^2c^2+4a^2c^2+2ab^2c-4a^2bc-4abc^2$$

$$=(ab)^2+(bc)^2+(-2ac)^2+2(ab)(bc)+2(bc)(-2ac)+2(-2ac)(ab)$$

$$=(ab+bc-2ac)^2$$

Hence the roots are rational.

9. For all values of k, prove that the roots of the equation

$x^2 - 2\left(k + \frac{1}{k}\right)x + 3 = 0, k \neq 0$ are real.

Solution:

$x^2 - 2\left(k + \frac{1}{k}\right)x + 3 = 0$

Here $a = 1, b = -2\left(k + \frac{1}{k}\right), c = 3$

Disc.

$= b^2 - 4ac$

$= \left[-2\left(k + \frac{1}{k}\right)\right]^2 - 4(1)(3)$

$= 4\left(k + \frac{1}{k}\right)^2 - 12$

$= 4\left[\left(k + \frac{1}{k}\right)^2 - 3\right]$

$= 4\left[k^2 + \frac{1}{k^2} + 2 - 3\right]$

$= 4\left[k^2 + \frac{1}{k^2} - 1\right] > 0$

Hence, the roots are real.

10. Show that the roots of the equation

$(b-c)x^2+(c-a)x+(a-b)=0$ are real.

Solution :

$(b-c)x^2+(c-a)x+(a-b)=0$

Here $a=(b-c), b=(c-a), c=(a-b)$

Disc.

$=b^2-4ac$

$=(c-a)^2-4(b-c)(a-b)$

$=c^2+a^2-2ac-4(ab-b^2-ac+bc)$

$=c^2+a^2-2ac-4ab+4b^2+4ac-4bc$

$=a^2+4b^2+c^2-4ab-4bc+2ac$

$=(a)^2+(-2b)^2+(c)^2+2(a)(-2b)+2(-2b)(c)+2(a)(c)$

$=(a-2b+c)^2>0$

Hence the roots of equation are real.

Plenary

Q. Show that the roots of the given equation are rational.

$(a + 2b)x^2 + 2(a + b + c)x + (a + 2c) = 0$

Solution

$$(a+2b)x^2+2(a+b+c)x+(a+2c)=0$$

Here $a=a+2b,\ b=2(a+b+c),\ c=a+2c$

Disc.

$$=b^2-4ac$$

$$=\left[2(a+b+c)\right]^2-4(a+2b)(a+2c)$$

$$=4(a+b+c)^2-4\left(a^2+2ac+2ab+4bc\right)$$

$$=4\left[a^2+b^2+c^2+2ab+2bc+2ca-a^2-2ac-2ab-4bc\right]$$

$$=4\left[b^2+c^2-2bc\right]$$

$$=4(b-c)^2$$

Hence the roots are rational.

Homework

Practice of Ex 2.1