

# WELCOME CLASS 10<sup>TH</sup> (SCIENCE) Theory of Quadratic Equations



#### Students will be able to: Solve questions related to discriminant

7. If the roots of the equation  $(c^2-ab)x^2-2(a^2-bc)x+(b^2-ac)=0$  are equal, then a = 0 or  $a^3 + b^3 + c^3 = 3abc$ Solution:  $(c^{2}-ab)x^{2}-2(a^{2}-bc)x+(b^{2}-ac)=0$ *Here*  $a = c^2 - ab, b = -2(a^2 - bc), c = b^2 - ac$ As the roots are equal, So Disc. = 0 $b^2 - 4ac = 0$  $\left[-2(a^2-bc)\right]^2 - 4(c^2-ab)(b^2-ac) = 0$  $4\left[\left(a^{2}-2a^{2}bc+b^{2}c^{2}\right)-\left(b^{2}c^{2}-ac^{3}-ab^{3}+a^{2}bc\right)\right]=0$  $a^{4} - 2a^{2}bc + b^{2}c^{2} - b^{2}c^{2} + ac^{3} + ab^{3} - a^{2}bc = 0$  $\Rightarrow a^4 + ab^3 + ac^3 - 3a^2bc = 0$  $a(a^3+b^3+c^3-3abc)=0$ Either a=0 or  $a^3 + b^3 + c^3 - 3abc$ Hence proved.

Activity

Q. Show that the roots of the given equation are rational.  $a(b-c)x^2 + b(c-a)x + c(a-b) = 0$ 

## Solution

$$a(b-c)x^{2}+b(c-a)x+c(a-b) = 0$$
Here  $a = a(b-c), b = b(c-a), c = c(a-b)$ 
Disc.  

$$= b^{2}-4ac$$

$$= [b(c-a)]^{2}-4[a(b-c)][c(a-b)]$$

$$= b^{2}(c-a)^{2}-4ac(ab-b^{2}-ac+bc)$$

$$= b^{2}c^{2}+a^{2}b^{2}-2ab^{2}c-4a^{2}bc+4ab^{2}c+4a^{2}c^{2}-4abc^{2}$$

$$= a^{2}b^{2}+b^{2}c^{2}+4a^{2}c^{2}+2ab^{2}c-4a^{2}bc-4abc^{2}$$

$$= (ab)^{2}+(bc)^{2}+(-2ac)^{2}+2(ab)(bc)+2(bc)(-2ac)+2(-2ac)(ab)$$

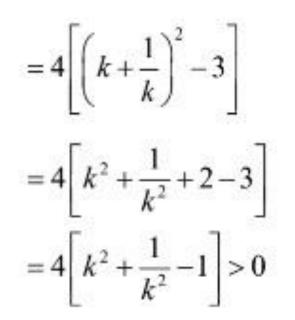
$$= (ab+bc-2ac)^{2}$$
Hence the roots are rational.

9. For all values of k, prove that the roots of the equation

$$x^2 - 2\left(k + \frac{1}{k}\right)x + 3 = 0, k \neq 0 \text{ are real.}$$

Solution:

 $x^2 - 2\left(k + \frac{1}{k}\right)x + 3 = 0$ Here  $a = 1, b = -2\left(k + \frac{1}{k}\right), c = 3$ Disc.  $=b^{2}-4ac$  $=\left[-2\left(k+\frac{1}{k}\right)\right]^2-4(1)(3)$  $=4\left(k+\frac{1}{k}\right)^{2}-12$ 



Hence, the roots are real.

#### 10. Show that the roots of the equation

$$(b-c)x^2 + (c-a)x + (a-b) = 0$$
 are real.

Solution :

$$(b-c)x^{2}+(c-a)x+(a-b)=0$$
  
Here  $a = (b-c), b = (c-a), c = (a-b)$ 

Disc.

$$= b^{2} - 4ac$$
  
=  $(c-a)^{2} - 4(b-c)(a-b)$   
=  $c^{2} + a^{2} - 2ac - 4(ab-b^{2} - ac + bc)$   
=  $c^{2} + a^{2} - 2ac - 4ab + 4b^{2} + 4ac - 4bc$   
=  $a^{2} + 4b^{2} + c^{2} - 4ab - 4bc + 2ac$   
=  $(a)^{2} + (-2b)^{2} + (c)^{2} + 2(a)(-2b) + 2(-2b)(c) + 2(a)(c)$   
=  $(a-2b+c)^{2} > 0$ 

Hence the roots of equation are real.



Q. Show that the roots of the given equation are rational.  $(a + 2b)x^2 + 2(a + b + c)x + (a + 2c) = 0$ 

## Solution

 $(a+2b)x^{2}+2(a+b+c)x+(a+2c)=0$ *Here* a = a + 2b, b = 2(a + b + c), c = a + 2cDisc.  $=b^{2}-4ac$  $= [2(a+b+c)]^2 - 4(a+2b)(a+2c)$  $= 4(a+b+c)^{2} - 4(a^{2}+2ac+2ab+4bc)$  $= 4 \left[ a^{2} + b^{2} + c^{2} + 2ab + 2bc + 2ca - a^{2} - 2ac - 2ab - 4bc \right]$  $=4\left[b^2+c^2-2bc\right]$  $=4(b-c)^{2}$ Hence the roots are rational.



Practice of Ex 2.1