Pakistan School
Kingdom of Bahrain

WELCOME CLASS 10TH (SCIENCE)

Theory of Quadratic Equations

Objectives

Students will be able to:

Find three cube roots of unity

2.2.1 The cube roots of unity.

Let a number x be the cube root of unity,

$$
\begin{aligned}
&i.e., && x = (1)^{1/3} \\
&\text{or} && x^3 = 1 \\
&\Rightarrow && x^3 - 1 = 0 \\
& && (x^3) - (1)^3 = 0 \\
& && (x-1)(x^2+x+1) = 0 \quad [\text{using } a^3 - b^3 = (a-b)(a^2+ab+b^2)]
\end{aligned}
$$

Either $x - 1 = 0$ or $x^2 + x + 1 = 0$

$\Rightarrow$ $x = 1$ or $x = \dfrac{-1 \pm \sqrt{(1)^2 - 4(1)(1)}}{2(1)}$

$$= \frac{-1 \pm \sqrt{1-4}}{2} = \frac{-1 \pm \sqrt{-3}}{2} = \frac{-1 \pm i\sqrt{3}}{2}$$

$\therefore$ Three cube roots of unity are

$1, \dfrac{-1+i\sqrt{3}}{2}$ and $\dfrac{-1-i\sqrt{3}}{2}$, where $i = \sqrt{-1}$.

(ii) The three cube roots of 8

Solution:

Let $x^3 = 8$

$x^3 - 8 = 0$

$(x)^3 - (2)^3 = 0$

$(x-2)(x^2+2x+4) = 0$

Either $x - 2 = 0$ *or* $x^2 + 2x + 4 = 0$

$x = 2$ *Here* $a = 1, b = 2, c = 4$

Using quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(4)}}{2(1)}$$

$$x = \frac{-2 \pm \sqrt{4-16}}{2}$$

$$x = \frac{-2 \pm \sqrt{-12}}{2}$$

$$x = \frac{-2 \pm 2\sqrt{-3}}{2}$$

$$x = \frac{2\left(-1 \pm i\sqrt{3}\right)}{2} \because i = \sqrt{-1}$$

$$x = \frac{2\left(-1 + i\sqrt{3}\right)}{2} \quad or \quad x = 2\left(-\frac{1 - i\sqrt{3}}{2}\right)$$

$$= 2\omega \qquad\qquad = 2\omega^2$$

Three cube roots of 8 are $2, 2\omega, 2\omega^2$

Activity

Q. Find the cube root of -27

Solution

Let $x^3 = -27$

$x^3 + 27 = 0$

$(x)^3 - (3)^3 = 0$

$(x+3)(x^2 - 3x + 9) = 0$

Either $x + 3 = 0$ *or* $x^2 - 3x + 9 = 0$

$x = -3$ *Here* $a = 1, b = -3, c = 9$

Using quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(9)}}{2(1)}$$

$$x = \frac{3 \pm \sqrt{9 - 36}}{2}$$

$$x = \frac{3 \pm \sqrt{-27}}{2}$$

$$x = \frac{3 \pm 3\sqrt{-3}}{2}$$

$$x = \frac{3(-1 \pm i\sqrt{3})}{2} \because i = \sqrt{-1}$$

$$x = \frac{3(1 + i\sqrt{3})}{2} \quad or \quad x = 3\left(-\frac{1 - i\sqrt{3}}{2}\right)$$

$$x = \frac{-3(-1 - i\sqrt{3})}{2} \quad or \quad x = -3\left(\frac{-1 + i\sqrt{3}}{2}\right)$$

$$= -3\omega^2 \qquad\qquad = -3\omega$$

Three cube roots of -27 are $-3, -3\omega, -3\omega^2$

Formulas

- $\omega^3 = 1$
- $\omega = \frac{1}{\omega^2}, \quad \omega^2 = \frac{1}{\omega}$
- $1 + \omega + \omega^2 = 0$

(i) $\left(1-\omega-\omega^2\right)^7$

Solution:

$$\begin{aligned}
\left(1-\omega-\omega^2\right)^7 &= \left[1-\left(\omega+\omega^2\right)\right]^7 \\
&= \left[1-1(-1)\right]^7 \quad \because \omega+\omega^2=-1 \\
&= (1+1)^7 \\
&= 2^7 = 128
\end{aligned}$$

(iii) $\left(9+4\omega+4\omega^2\right)^3$

Solution:

$$\begin{aligned}\left(9+4\omega+4\omega^2\right)^3 &= \left[9+4\left(\omega+\omega^2\right)\right]^3\\ &= \left[9+4(-1)\right]^3 \because \omega+\omega^2=-1\\ &= (9-4)^3\\ &= 5^3 = 125\end{aligned}$$

Plenary

Q. Evaluate

$$\left(1-3\omega-3\omega^2\right)^5$$

Solution

$$
\begin{aligned}
\left(1-3\omega-3\omega^2\right)^5 &= \left[1-3\left(\omega+\omega^2\right)\right]^5 \\
&= \left[1-3(-1)\right]^5 \quad \because \omega+\omega^2=-1 \\
&= (1+3)^5 \\
&= 4^5 = 1024
\end{aligned}
$$

Homework

Practice of Ex 2.2 Q1 part 4 and Q2 part (iv)