Pakistan School
Kingdom of Bahrain

WELCOME CLASS 10TH (SCIENCE)

Theory of Quadratic Equations

Objectives

Students will be able to:

Simplify the questions related to cube root of unity

(v) $\left(-1+\sqrt{-3}\right)^{6}+\left(-1-\sqrt{-3}\right)^{6}$

Solution:

$$\left(-1+\sqrt{-3}\right)^{6}+\left(-1-\sqrt{-3}\right)^{6}$$

$$=\left(2\omega\right)^{6}+\left(2\omega^{2}\right)^{6} \qquad \because \omega=\frac{-1+\sqrt{-3}}{2} \ \ and \ \ \omega^{2}=\frac{-1-1\sqrt{-3}}{2}$$

$$=2^{6}\left(\omega^{6}\right)+2^{6}\left(\omega^{12}\right) \qquad 2\omega=-1+\sqrt{-3} \ \ and \ \ 2\omega^{2}=-1-1\sqrt{-3}$$

$$=2^{6}\left[\left(\omega^{3}\right)^{2}\right]+2^{6}\left[\left(\omega^{3}\right)^{4}\right]$$

$$=2^{6}\left[(1)^{2}\right]+2^{6}\left[(1)^{4}\right] \qquad \because \omega^{3}=1$$

$$=2^{6}\left[1+1\right]$$

$$=2^{6}.2=2^{6+1}=2^{7}$$

$$=128$$

(vii) $\omega^{37}+\omega^{38}-5$

Solution:

$$\omega^{37}+\omega^{38}-5$$
$$=\omega^{37}+\omega^{38}-5$$
$$=\omega^{36}.\omega+\omega^{36}.\omega^{2}-5$$
$$=\left(\omega^{3}\right)^{12}.\omega+\left(\omega^{3}\right)^{12}.\omega^{2}-5$$
$$=(1)^{12}.\omega+(1)^{12}.\omega^{2}-5 \qquad \because \omega^{3}=1$$
$$=\omega+\omega^{2}-5$$
$$=-1-5 \qquad \because \omega+\omega^{2}=-1$$
$$=-6$$

3. Prove that $x^3+y^3=(x+y)(x+\omega y)(x+\omega^2 y)$.

Solution:

$$x^3+y^3=(x+y)(x+\omega y)(x+\omega^2 y)$$

$$\begin{aligned}
R.H.S &= (x+y)(x+\omega y)(x+\omega^2 y)\\
&= (x+y)\left[x(x+\omega^2 y)+\omega y(\omega+\omega^2 y)\right]\\
&= (x+y)\left[x^2+\omega^2 xy+\omega xy+\omega^3 y^2\right]\\
&= (x+y)\left[x^2+(\omega^2+\omega)xy+(1)y^2\right] \quad \because \omega^3=1\\
&= (x+y)\left[x^2+(-1)xy+y^2\right] \quad \because \omega^2+\omega=-1\\
&= (x+y)(x^2-xy+y^2)\\
&= L.H.S
\end{aligned}$$

Hence Proved.

5. Prove that $(i+\omega)(i+\omega^2)(i+\omega^4)(i+\omega^8).....2\text{n factors}=1$

Solution:

$$L.H.S=(i+\omega)(i+\omega^2)(i+\omega^4)(i+\omega^8).....2\text{n factors}$$

$$=(i+\omega)(i+\omega^2)(i+\omega^3.\omega)(i+\omega^2.\omega^6).....2\text{n factors}$$

$$=(i+\omega)(i+\omega^2)(i+\omega^3.\omega)\left(i+\omega^2.(\omega^3)^2\right).....2\text{n factors}$$

$$=(i+\omega)(i+\omega^2)(i+(1)\omega)\left(i+\omega^2(1)^2\right).....2\text{n factors} \quad \because \omega^3=1$$

$$=(i+\omega)(i+\omega^2)(i+\omega)(i+\omega^2).....2\text{n factors}$$

$$=(-\omega^2)(i+\omega^2)(-\omega^2)(i+\omega^2).....2\text{n factors} \qquad \because 1+\omega=-\omega^2$$

$$=\left[(-\omega^2)(-\omega)\right]\left[(-\omega^2)(-\omega)\right].....\text{n factors} \qquad \because 1+\omega^2=-\omega$$

$$=\left[\omega^3\right]\left[\omega^3\right].....\text{n factors}$$

$$=(1)(1).....\text{n factors} \qquad \because \omega^3=1$$

$$=(1)^n$$

$$=1$$

Hence Proved.

$$=R.H.S$$

Activity

Q. Simplify

$$\left(\frac{-1+\sqrt{-3}}{2}\right)^9+\left(\frac{-1-\sqrt{-3}}{2}\right)^9$$

Solution

$$\left(\frac{-1+\sqrt{-3}}{2}\right)^9+\left(\frac{-1-\sqrt{-3}}{2}\right)^9$$

$$=\omega^9+\left(2\omega^2\right)^9 \qquad \because \omega=\frac{-1+\sqrt{-3}}{2} \quad and \quad \omega^2=\frac{-1-1\sqrt{-3}}{2}$$

$$=\omega^9+\omega^{18}$$

$$=\left(\omega^3\right)^3+\omega^{18}$$

$$=\left(\omega^3\right)^3+\left(\omega^3\right)^6=(1)^3+(1)^6 \qquad \because \omega^3=1$$

$$=1+1=2$$

Homework

Ex 2.2 Q4