Pakistan School
Kingdom of Bahrain

WELCOME CLASS 10TH (SCIENCE)

Theory of Quadratic Equations

Objectives

Students will be able to:

Find sum and product of roots of the given equation

2.3 Roots and co-efficients of a quadratic equation.

We know that $\frac{-b+\sqrt{b^2-4ac}}{2a}$ and $\frac{-b-\sqrt{b^2-4ac}}{2a}$ are roots of the equation $ax^2 + bx + c = 0$ where a, b are coefficients of x^2 and x respectively. While c is the constant term.

2.3.1 Relation between roots and co-efficients of a quadratic equation.

If $\alpha = \frac{-b+\sqrt{b^2-4ac}}{2a}$ and $\beta = \frac{-b-\sqrt{b^2-4ac}}{2a}$,

then we can find the sum and the product of the roots as follows.

Sum of the roots $= \alpha + \beta$

$$= \frac{-b+\sqrt{b^2-4ac}}{2a} + \frac{-b-\sqrt{b^2-4ac}}{2a}$$

$$= \frac{-b+\sqrt{b^2-4ac}-b-\sqrt{b^2-4ac}}{2a} = \frac{-2b}{2a} = -\frac{b}{a}$$

Product of the roots $= \alpha\beta$

$$= \left(\frac{-b+\sqrt{b^2-4ac}}{2a}\right)\left(\frac{-b-\sqrt{b^2-4ac}}{2a}\right)$$

$$= \frac{(-b)^2-(\sqrt{b^2-4ac})^2}{4a^2} = \frac{b^2-(b^2-4ac)}{4a^2}$$

$$= \frac{b^2-b^2+4ac}{4a^2} = \frac{4ac}{4a^2} = \frac{c}{a}.$$

If we denote the sum of roots and product of roots by S and P respectively, then

$$S = -\frac{b}{a} = -\frac{\text{Co-efficient of } x}{\text{Co-efficient of } x^2}$$

and $$P = \frac{c}{a} = \frac{\text{Constant term}}{\text{Co-efficient of } x^2}.$$

1. Without solving, find the sum and the product of the following quadratic equations.

(i) $x^2 - 5x + 3 = 0$

Solution:

$x^2 - 5x + 3 = 0$

Here a=1, b=-5, c=3

Let α and β be the roots of the given equation

Then Sum of roots = $\alpha + \beta = -\frac{b}{a} = -\frac{(-5)}{1} = 5$

And product of roots = $\alpha\beta = \frac{c}{a} = \frac{3}{1} = 3$

Activity

1. Without solving, find the sum and the product of the following quadratic equations.

(ii) $3x^2 + 7x - 11 = 0$

Solution

$3x^2 + 7x - 11 = 0$

Here a=3, b=7, c=-11

Let α and β be the roots of the given equation

Then Sum of roots = $\alpha + \beta = -\frac{b}{a} = -\frac{7}{3}$

And product of roots = $\alpha\beta = \frac{c}{a} = -\frac{11}{3}$

2. Find the value of k, if

(i) Sum of the roots of the equation 2kx2-3x+4k=0 is twice the product of the roots.

Solution:

$2kx^2 - 3x + 4k = 0$

Here a=2k, b=-3, c=4k

Let α and β be the roots of the given equation

Then Sum of roots = $\alpha + \beta = -\frac{b}{a} = -\frac{(-3)}{2k} = \frac{3}{2k}$

And product of roots = $\alpha\beta = \frac{c}{a} = \frac{4k}{2k} = 2$

As sum of the roots is twice the product of the roots, so

$\alpha + \beta = 2\alpha\beta$

$\frac{3}{2k} = 2(2)$

$\frac{3}{2k} = 4$

$or\ k = \frac{3}{8}$

Activity

2. Find the value of k, if

(ii) Sum of the roots of the equation $x^2+(3k-7)x+5k=0$ **is** $\frac{3}{2}$ **times the product of the roots.**

Solution

$x^2+(3k-7)x+5k=0$

Here a=1, b=$3k-7$, c=5k

Let α and β be the roots of the given equation

Then Sum of roots = $\alpha+\beta=-\frac{b}{a}=-\frac{3k-7}{1}=-3k+7$

And product of roots = $\alpha\beta=\frac{c}{a}=\frac{5k}{1}=5k$

As sum of the roots is $\frac{3}{2}$ of the product of the roots, so

$\alpha+\beta=\frac{3}{2}\alpha\beta$

$-3k+7=\frac{3}{2}(5k)$

$-3k+7=\frac{15k}{2}$

$-3k-\frac{15k}{2}=-7$

$\frac{-6k-15k}{2}=-7$

$\frac{-21k}{2}=-7$

$k=(-7)\left(-\frac{2}{21}\right)$

$k=\frac{2}{3}$

Homework

Ex 2.3 Q1 (iii,iv,v,vi)