Pakistan School
Kingdom of Bahrain

WELCOME CLASS 10TH (SCIENCE)

Quadratic Equations

Objectives

Students will be able to:

Solve equations which are convertible to quadratic equations

1.4 Equations reducible to quadratic form

We now discuss different types of equations, which can be reduced to a quadratic equation by some proper substitution.

Type (i) The equations of the type $ax^4 + bx^2 + c = 0$

Replacing $x^2 = y$ in equation $ax^4 + bx^2 + c = 0$,

we get a quadratic equation in y.

Example 1: Solve the equation $x^4 - 13x^2 + 36 = 0$.

Solution: $x^4 - 13x^2 + 36 = 0$ (i)

Let $x^2 = y$. Then $x^4 = y^2$

Equation (i) becomes

$y^2 - 13y + 36 = 0$ which can be factorized as

$y^2 - 9y - 4y + 36 = 0$

$y(y - 9) - 4(y - 9) = 0$

$(y - 9)(y - 4) = 0$

Either $y - 9 = 0$ or $y - 4 = 0$, that is,

$y = 9$ or $y = 4$

Put $y = x^2$

$x^2 = 9$ or $x^2 = 4$

$\Rightarrow$ $x = \pm 3$ or $x = \pm 2$

$\therefore$ The solution set is $\{\pm 2, \pm 3\}$

Q. Solve the following equations

(1) $2x^4 - 11x^2 + 5 = 0$

$Solution: 2x^4 - 11x^2 + 5 = 0$

$Let \quad x^2 = y \Rightarrow (x^2)^2 = y^2$

$x^4 = y^2$

$Put \quad x^2 = y \quad and \quad x^4 = y^2$

$2x^4 - 11x^2 + 5 = 0$

$2y^2 - 11y + 5 = 0$

$2y^2 - 10y - y + 5 = 0$

$2y(y-5) - 1(y-5)$

$(y-5)(2y-1)$

$y - 5 = 0 \quad or \quad 2y - 1 = 0$

$y = 5 \quad or \quad 2y = 1$

$y = 5 \quad or \quad y = \frac{1}{2}$

$Put\ in \quad x^2 = y$

$x^2 = 5 \quad or \quad x^2 = \frac{1}{2}$

$\sqrt{x^2} = \pm\sqrt{5} \qquad \sqrt{x^2} = \pm\sqrt{\frac{1}{2}}$

$x = \pm\sqrt{5} \qquad x = \pm\frac{1}{\sqrt{2}}$

$Solution\ Set\ is \left\{\pm\frac{1}{\sqrt{2}}, \pm\sqrt{5}\right\}$

(5) $3x^{-2} + 5 = 8x^{-1}$

$Solution: 3x^{-2} + 5 = 8x^{-1} \ldots\ldots\ldots(1)$

$x^{-1} = y \qquad x^{-2} = y^2 \quad \ldots\ldots(2)$

$Put\ x^{-1} = y\ \ and\ \ x^{-2} = y^2\ in\ eq\,(1)$

$3y^2 + 5 = 8y$

$3y^2 - 8y + 5 = 0$

$3y^2 - 3y - 5y + 5 = 0$

$3y(y-1) - 5(y-1) = 0$

$(3y-5)(y-1) = 0$

$3y - 5 = 0 \qquad y - 1 = 0$

$3y = 5 \qquad y = 1$

$y = \frac{5}{3} \qquad y = 1$

$From\ eq\,(2)\ put\ y = x^{-1}$

$x^{-1} = \frac{5}{3} \qquad x^{-1} = 1$

$\frac{1}{x} = \frac{5}{3} \qquad \frac{1}{x} = 1$

$x = \frac{3}{5} \qquad x = 1$

$S.Set = \left\{\frac{3}{5}, 1\right\}$

Plenary

Q. Solve the following equation

$2x^4 = 9x^2 - 4$

Solution

$2x^4 = 9x^2 - 4$

Solution: $2x^4 = 9x^2 - 4$

$2x^4 - 9x^2 + 4 = 0 \quad .....(1)$

Let $x^2 = y \qquad .....(2)$

Taking square on both sides

$(x^2)^2 = y^2$

$x^4 = y^2$

Put $x^2 = y$ *and* $x^4 = y^2$ *in eq*(1)

$2y^2 - 9y + 4 = 0$

$2y^2 - 8y - 1y + 4 = 0$

$2y(y-4) - 1(y-4) = 0$

$(y-4)(2y-1) = 0$

$y - 4 = 0 \qquad 2y - 1 = 0$

$y = 4 \qquad 2y = 1$

$y = 4 \qquad y = \frac{1}{2}$

Put $y = x^2$

$x^2 = 4 \qquad x^2 = \frac{1}{2}$

$\sqrt{x^2} = \pm\sqrt{4} \qquad \sqrt{x^2} = \sqrt{\frac{1}{2}}$

$x = \pm 2 \qquad x = \pm\frac{1}{\sqrt{2}}$

Solution set is $\left\{\pm 2, \pm\frac{1}{\sqrt{2}}\right\}$

Homework

Ex 1.3 Remaining parts