Pakistan School
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Dimensions
Class 11

Dimension

- <u>Dimensions</u>
- The powers positive or negative to which the fundamental units of first system must be raised to give the unit of a given physical quantity (derived unit) are called dimensions of that quantity.
- <u>Dimensions of physical quantities</u>
- In physics, the word 'dimension represents the nature of a physical quantity, For example, different quantities such as length, breadth, diameter, depth or wave length which are measured in metre denote the same dimension i.e.
- length [L]
- similarly the mass [M]
- and time dimension [T] respectively.
- The dimension of a derived quantity are a product of the dimensions of the base quantities from which the quantity is derived. For example, area defined as (length x length) is dimensionally Area = $[L^2]$. Speed v is defined a distance covered per unit time.

Examples

For Velocity

$$v = \text{distance}/\text{time}$$

$$= [L] \div [T]$$

$$= [L][T^{-1}]$$

$$= [LT^{-1}]$$

Some Terms Used with Dimensions

- Dimensional variables: Those physical quantities that have dimensions but,variable magnitude are dimensional variable quantities.
- Examples: (1) Force (2) Energy (3) Acceleration etc.
- Dimensional constants: Those physical dimensional quantities which arc
- constant in magnitude are dimensionally constant quantity.

  Examples: (1) Speed of light in vacuum (2) Planks constant (3) Gravitational constant. (4) Ideal gas constant R etc.
- Dimensionless variable: Those physical dimensionless quantities having variables magnitude are dimensionless variables.

  Examples: (1) Plane angle (2) Solid angle (3) Strain (4) Co-efficient of friction etc.
- Dimensionless constants: Those physical quantities which have no dimensions but are constant are called dimensionless constants.

  Examples (1) Pure number1, 2, 3, 4... (2) $\pi = 22/7$

Advantages of Dimension

- With dimensional analysis we can check the homogeneity of a physical equation. A homogeneous equation may or may not be correct.
- Derive a possible formula for a Physical problem

Examples

For Velocity

$v = d/t = [L] \div [T] = [L][T^{-1}] = [LT^{-1}]$

Similarly the dimension of acceleration are

$a = d/t/t = [L] \div [T^2] = [LT^{-2}]$

and that of force are

$[F] = [m]\,[a] = [M][LT^{-2}] = [MLT^{-2}]$

Examples from book (page 17)

$$S = vit + \frac{1}{2}at^2 \qquad 1.4$$

The dimensions of S in LHS = [L]

$$\text{And on RHS} = vit + \frac{1}{2}at^2 = \frac{LT}{T} + \frac{LT^2}{2T^2}$$

$$RHS = L + \frac{1}{2}L = \frac{3}{2}L = \frac{3}{2}[L] \qquad 1.5$$

From 1.4 and 1.5 it is clear that the dimensions are same on both sides, thus the equation is correct dimensionally. This is called principle of homogeneity.

- **$S=V_it+at^2$ ---------- 1.4**
- **The dimension of S in LHS =[L]**
- **And on RHS =Vit + at² = $\frac{LT}{T} + \frac{LT^2}{2T^2}$**
- **RHS = L+ (1/2)L =(3/2) L = (3/2) [L] ----------------1.5**

Examples from book (page 18)

Example 1.6 The energy of a photon E = hf, Find the dimensions of Planck's constant h where f is the frequency.

Solution:

Planck's constant $h = \frac{E}{f}$

Now dimensions of $E = [M^1L^2T^{-2}]$ and $f = [T^{-1}]$

Dimensions of $h = \frac{[M^1L^2T^{-2}]}{[T^{-1}]}$

$= [M^1L^2T^{-1}]$

- **Planks Constant h =** $\frac{E}{T^{-1}}$
- Where E = mgh = (kg)(m/s²)(m)
- $= [M][\frac{L}{T^2}][L] = [M^1L^2T^{-2}]$
- Dimension for $h = [M^1L^2T^{-2}]/[T^{-1}] = \frac{[M^1L^2T^{-2}]}{[T^{-1}]}$
- $= [M^1L^2T^{-1}]$

Example 1.5 page 17-18

- The time period of a simple pendulum possibly depends upon:
- Length of the pendulum
- Mass of the bob
- Acceleration due to gravity
- Angular displacement

- Suppose
- T= time period of the pendulum
- G=acceleration due to gravity
- L=length of pendulum
- θ = angle with the mean position
- T is directly proportion to some power of l, g and m i.e
-
- $T \propto l^a$, $T \propto g^b$, $T \propto m^c$
- $T \propto l^a \; g^b \; m^c$
- $T = k \; l^a \; g^b \; m^c$ -----------------A
- $[M^0L^0T^1] = k \, [L]^a [LT^{-2}]^b [M]^c$
- $[M^0L^0T^1] = k \, [L]^a [L^b T^{-2b}] \, [M]^c$
- $[M^0L^0T^1] = k \, [L^a L^b][T^{-2b}] \, [M]^c$
- $[M^0L^0T^1] = k \, [L^a L^b][T^{-2b}] \, [M]^c$
- $[M^0L^0T^1] = k \, [L^{a+b}][T^{-2b}] \, [M]^c$
- $[M^0L^0T^1] = k \, [L^{a+b}][T^{-2b}] \, [M]^c$
- Comparing the power of M, L and T,
- we get $c=0$, $a+b=0$ and $-2b=1$ or $b=-1/2$
- Find "a", we have
- $a+b=0$
- $a+(1/2)=0 \quad \rightarrow a=-1/2$
- Now Pitting values in Equation A, we get
- $T = k \; l^{1/2} \; g^{-1/2} \; m^0$ where $m^0 = 1$
- $T = k \; l^{1/2} / g^{1/2}$

$$T = k \frac{l^{\frac{1}{2}}}{g^{\frac{1}{2}}}$$

$$T = 2\pi \frac{l^{\frac{1}{2}}}{g^{\frac{1}{2}}}$$

$$T = 2\pi \sqrt{\frac{l}{g}}$$

Example from the book Page 17-18

Example 1.5 Deduce relation for time period of simple pendulum.

Solution:

Time period of simple pendulum possibly depends upon

(i) length of pendulum. (ii) mass of bob

(iii) acceleration due to gravity. (iv) angular displacement θ.

Suppose T = time period of pendulum

g = acceleration due to gravity

ℓ = length of pendulum

θ = angle with mean position

Suppose T is directly proportional to some powers of g, ℓ, m, i.e.

$T \propto \ell^a$, $T \propto g^b$ and $T \propto m^c$

Combining the above results we get

$T \propto \ell^a g^b m^c \quad \Rightarrow \quad T = k\, \ell^a g^b m^c$

Where k is constant of proportionality substituting dimensions of various quantities is

$$T = k\ell^a g^b m^c \qquad \text{A}$$

$$[M^oL^oT^1] = k[L]^a [LT^{-2}]^b M^c$$
$$[M^oL^oT^1] = k[L^a L^b T^{-2} M^c$$
$$\Rightarrow [M^oL^oT^1] = k[L^{a+b}T^{-2b}M^c$$
$$\Rightarrow [M^oL^oT^1] = k[L]^{a+b} [T]^{-2b} [M]^c$$

Comparing the powers of M, L and T, we get $C = 0$, $a + b=0$ and $-2b=1$ or $b=-\frac{1}{2}$

To find "a" we have

$$a + b = 0$$
$$a - \frac{1}{2} = 0 \Rightarrow a = \frac{1}{2}$$

Now putting values in equation (A) we get

$$T = k\ell^{\frac{1}{2}} \; g^{-\frac{1}{2}} \; m^o \qquad \text{where } m^o = 1$$

$$T = k\frac{\ell^{\frac{1}{2}}}{g^{\frac{1}{2}}} = k\sqrt{\frac{\ell}{g}}$$

Where the value of k found experimentally is 2π

Thus $T = 2\pi\sqrt{\frac{\ell}{g}}$ which is the equation for time period of simple pendulum

Closure :Planery:Question?

- What is the Dimension for Area?

- Area= Length x Length
- =[L]x[L]
- $=[L^2]$
- $=[L]^2$

Home Work

- Attempt any question from the given assignment on the website.
- Or
- Create dimension for different physical quanties