Pakistan School
Kingdom of Bahrain

Rules of the Class:

- 1) Always be on time for all your classes
- 2)Always Respect your all Class fellows.
- 3)Do not create any disturbance.
- 4)Raise hand if you have any question or you wish to answer any question.
- 5)Pay attention to your teacher.
- 6)Please, Enter into the class with your actual Name and CPR number.
- 7)Always follow your Time Table.

Engaging Starter

Physical Quantities Having

Both Magnitude and Direction

are known as ___________

Grade 11th ” Physics ”

Unit: 2 “Vectors and Equilibrium”

Topic:2.1 “Products of Vectors”

Learning Objectives: By the end of the session, students will be able to:

1) **Define Scalar and vector products.**

2) **Differentiate Dot and cross products of vectors.**

Product of vectors

Scalar Product

When the product of two vectors result a scalar quantity, such product of vectors is called scalar product or dot product.

Vector quantity • Vector quantity = Scalar quantity

Force • displacement = Work

$\vec{F} \cdot \vec{S} = W$

Force • Velocity = Power

$\vec{F} \cdot \vec{V} = P$

Vector Product

When the product of two vectors result a vector quantity, such product of vectors is called vector product or cross product.

Vector quantity × Vector quantity = Vector quantity

Moment arm × Force = Torque

$\vec{r} \times \vec{F} = \vec{\tau}$

Position vector × linear momentum = angular momentum

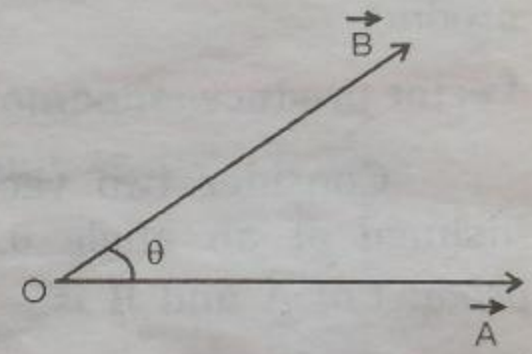
$\vec{r} \times \vec{p} = \vec{L}$

Scalar Product:(Dot Product)

Scalar product

Consider two vectors $\vec{A}$ and $\vec{B}$ which are inclined at an angle θ. By definition the scalar product of $\vec{A}$ and $\vec{B}$ is

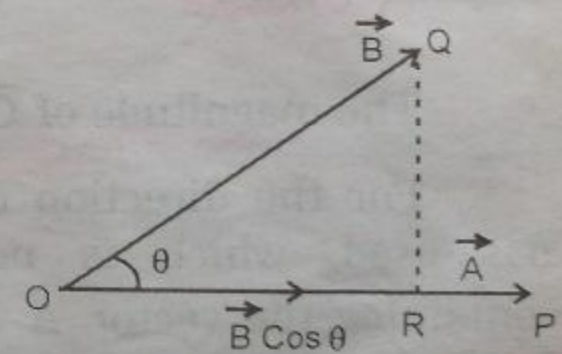
$$\vec{A} \bullet \vec{B} = A\,B\,\text{Cos}\theta \quad \text{———— (i)}$$



From Q draw QR perpendicular on OP. In triangle OQR when the vector $\vec{B}$ is resolved, the component of $\vec{B}$ in the direction of $\vec{A}$ is Bcosθ.

$$\vec{A} \bullet \vec{B} = A\ B\,\text{Cos}\theta$$

$\vec{A} \bullet \vec{B} =$ (magnitude of $\vec{A}$) times (the magnitude of the component of $\vec{B}$ in the direction of $\vec{A}$).

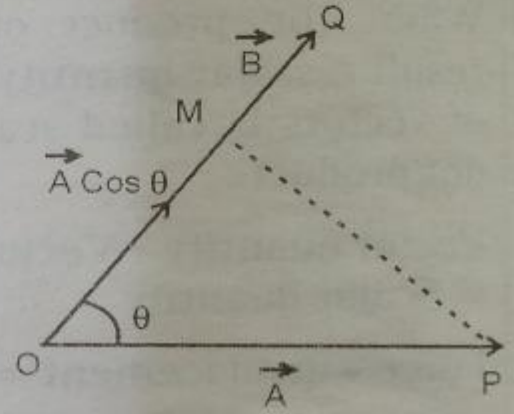


The component of $\vec{B}$ in the direction of $\vec{A}$ is called projection of $\vec{B}$ on $\vec{A}$. $\vec{A} \bullet \vec{B} =$ (magnitude of $\vec{A}$) times (projection of $\vec{B}$ on $\vec{A}$)

Scalar Product:(Dot Product)

Similarly from P draw PM perpendicular on OQ. In triangle OPM when the vector $\vec{A}$ is resolved, its component in the direction of $\vec{B}$ is ACosθ

$\vec{B} \bullet \vec{A} = B\,A\,\text{Cos}\theta$ ———— (ii)



$\vec{B} \bullet \vec{A} =$ (magnitude of B) times (magnitude of component of $\vec{A}$ in the direction of $\vec{B}$)

$\vec{B} \bullet \vec{A} =$ (magnitude of $\vec{B}$) times (projection of $\vec{A}$ on $\vec{B}$)

AB Cosθ and BA Cosθ are both scalar quantities which are equal in magnitude

$AB\,\text{Cos}\theta = BA\,\text{Cos}\theta$

putting eq. (i) and (ii)

$\vec{A} \bullet \vec{B} = \vec{B} \bullet \vec{A}$

This property of the scalar product shows that the scalar product is commutative. According to the commutative property if the order of the vector in scalar product be changed, it will have no effect on the scalar product.

Examples:

1. WORK

For force making are angle θ with direction of the distance through which body moves, the work done is given by; $W = \vec{F}.\vec{d}$

2. POWER

Power, a scalar product of force and velocity, given by;

$$P = \vec{F}.\vec{V}$$

3. KINETIC ENERGY

Kinetic energy of an object of mass "m" also involves the dot product of velocity vector with itself i.e. $K.E = \frac{1}{2} m \vec{V}.\vec{V}$

4. POTENTIAL ENERGY

Potential energy of a body is the scalar product of gravitational field and displacement

$$P.E = m\, \vec{g}.\vec{h}$$

Physical Significance of Dot Product

PHYSICAL SIGNIFICANCE OF DOT PRODUCT

1. Self Product

When the length of vectors is defined, it is possible to find the magnitude of a vector by a dot product by taking the square root of the dot product of a vector by itself

i.e., $\vec{V}.\vec{V} = V^2$

2. Angle Between Two Vectors

Dot product is also used to find the angle between any two vectors. For angle between two any two vectors $\vec{A}$ and $\vec{B}$;

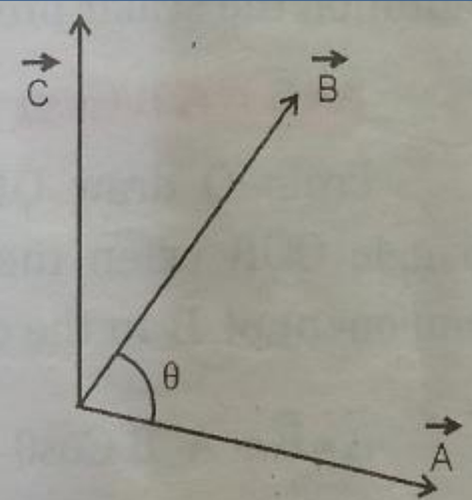
$$\theta = \text{Cos}^{-1}\left(\frac{\vec{A}.\vec{B}}{|A||B|}\right)$$

Vector Product:(Cross product)

Vector product

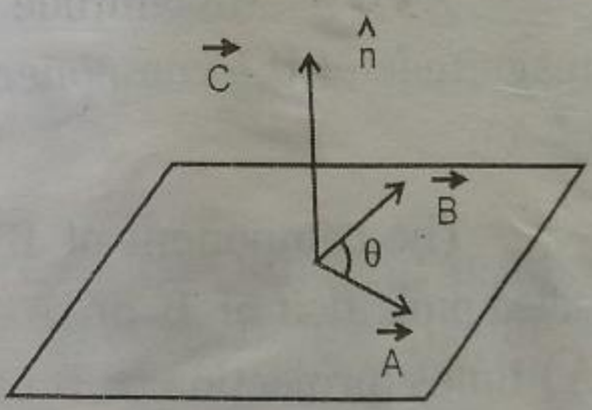
Consider two vectors $\vec{A}$ and $\vec{B}$ which are inclined at an angle θ. By definition the vector product of $\vec{A}$ and $\vec{B}$ is

$$\vec{A} \times \vec{B} = \vec{C} \qquad \text{———— (i)}$$



The magnitude of $\vec{C}$ is $C = AB\,\text{Sin}\theta$

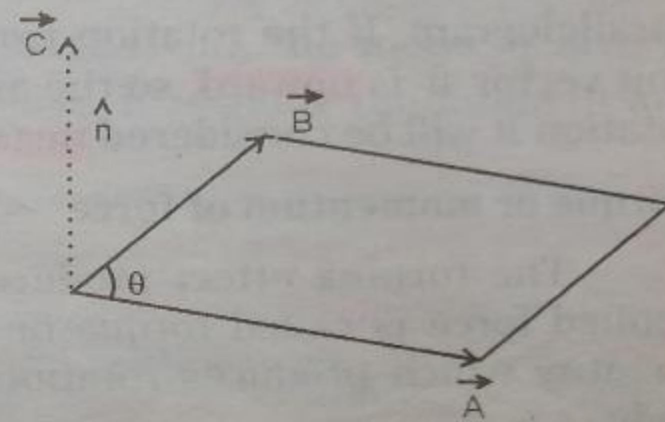
For the direction of $\vec{C}$, the unit vector $\hat{n}$ is used, which is normal to the plane containing the vector $\vec{A}$ and $\vec{B}$. The vector $\vec{C}$ in magnitude and direction is



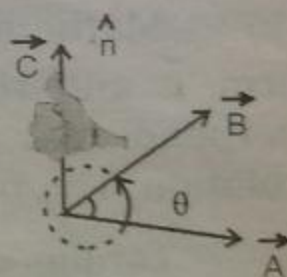
$\vec{C} = AB\,\text{Sin}\theta \bullet \hat{n}$ putting in eq. (i)

Vector Product

A is the magnitude of vector $\overline{A}$, B Sinθ is the component of vector $\overline{B}$ perpendicular to vector $\overline{A}$ and $\hat{n}$ is the unit vector perpendicular to the plane determined by vector $\overline{A}$ and vector $\overline{B}$.



For the direction of vector $\overline{C}$ the right hand rule is used, according to which if the 1st vector be rotated towards and 2nd vector through a small angle, the stretched thumb points in direction of $\overline{C}$. For anticlockwise rotation of vectors, the direction of the vector $\overline{C}$ i.e. unit vector $\hat{n}$ is upward which is considered positive. But for clockwise rotation of vectors, the direction of $\overline{C}$ i.e. unit vector $\hat{n}$ is downward which is considered negative.

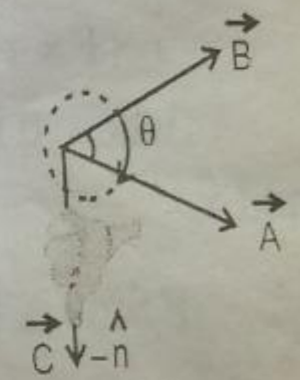


$$\overline{B} \times \overline{A} = B\,A\,\text{Sin}\,\theta\,(-\hat{n})$$

$$\overline{B} \times \overline{A} = -B\,A\,\text{Sin}\,\theta\,\hat{n} \qquad \text{———— (iii)}$$

Vector Product

B is magnitude of vector $\vec{B}$, A Sinθ is the component of $\vec{A}$ perpendicular to vector $\vec{B}$ and $\hat{n}$ is the unit vector perpendicular to the plane containing $\vec{A}$ and $\vec{B}$



From eq. (ii) and (iii)

$$AB \, Sin\theta \bullet \hat{n} = -B \, A \, Sin\,\theta \, \hat{n}$$

$$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$$

This eq. shows that the vector product is not commutative. The vector products do not obey the commutative law.

Examples:

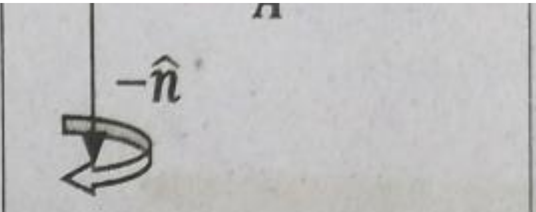


Figure 2.15

1. Torque

The cross product of force and moment arm is called as torque. i.e. $\vec{\tau} = \vec{r} \times \vec{F}$

2. Angular Momentum

The cross product of position vector and linear momentum gives angular momentum. i.e. $\vec{L} = \vec{r} \times \vec{P}$

3. Linear Velocity

It is vector quantity defined by cross product of angular velocity $\vec{\omega}$ with the moment arm $\vec{r}$. Mathematically $\vec{V} = \vec{\omega} \times \vec{r}$

4. Linear Acceleration

The cross product of angular acceleration "α" and moment arm "r" gives linear acceleration". Mathematically $\vec{a} = \vec{\alpha} \times \vec{r}$

Physical Significance of Dot Product

PHYSICAL SIGNIFICANCE OF VECTOR PRODUCT

1. Magnitude of Cross Product

The magnitude of $\vec{A} \times \vec{B}$ i.e. AB sin θ, gives the area of the plane determined by $\vec{A}$ & $\vec{B}$.

The unit vector $\hat{n}$ gives the direction of the area of the plane.

An area is considered positive if it lies to the left of the describing vectors, otherwise will be considered negative.

2. Angle Between Two Vectors

Cross product is also used to find the angle between any two vectors. For angle between two any two vectors $\vec{A}$ and $\vec{B}$.

$$\theta = \text{Sin}^{-1}\left(\frac{|\vec{A} \times \vec{B}|}{|\vec{A}||\vec{B}|}\right)$$

Closure

a) Define Dot product of vector.

b) Define Cross product of vector.

c) Name any three examples of Dot product.

d) Name any three examples of Cross product.

Home Work

Assignment is given in uploaded notes.

Thank you…..